MATH 2050B Tutorial 2

October 5, 2016

Exercise 1. Evaluate the following limits by definition

$$\lim_{n\to\infty} \frac{5n^2+2n+1}{3n^2+n+2}, \tag{1}$$

$$\lim_{n\to\infty} \frac{5n^2+2n+1}{3n^2-n-1}. \tag{2}$$

Exercise 2. Let A be a nonempty bounded above subset of $\mathbb{R}$, and let $\sup(A) = \alpha \in \mathbb{R}$.

1. Construct a monotone increasing sequence (a_n) in A converging to α.
2. Suppose further $\alpha \notin A$, construct a **strictly** increasing sequence (a_n) in A converging to α.

Exercise 3. For any fixed $a > 0$, show that $\lim_{n\to\infty} \frac{a^n}{n!} = 0$.

Exercise 4. For any fixed $p \in \mathbb{N}$, and $b \in \mathbb{R}$ satisfying $0 < b < 1$, show that $\lim_{n\to\infty} n^p b^n = 0$.

Exercise 5. (Ratio Test) Let (x_n) be a sequence of positive real numbers. Suppose

$$\lim_{n\to\infty} \frac{x_{n+1}}{x_n} = L, \tag{3}$$

where L is a non-negative real number.

(a) If $0 \le L < 1$, show that $\lim_{n\to\infty} x_n = 0$.

(b) If $L > 1$, show that (x_n) is divergent.

(c) If $L = 1$, show this method can not be used as a test for convergence. That is, there exists a convergence sequence (x_n) such that (3) holds. There also exists a divergent sequence (x_n) such that (3) holds.

Exercise 6. (Root Test) Let (x_n) be a sequence of positive real numbers. Suppose

$$\lim_{n\to\infty} \sqrt[n]{x_n} = L,$$

where L is a non-negative real number.

(a) If $0 \le L < 1$, show that $\lim_{n\to\infty} x_n = 0$.

(b) If $L > 1$, show that (x_n) is divergent.

(c) What happens if $L = 1$?